MT 1800 Calculus I

Module III, CA 1: The Derivative of $f \cdot g$ (The Product Rule)

Name: _____

Purpose: To develop a rule for calculating the symbolic formula for the derivative of a product of two functions.

Procedure: Work on the following activity together in pairs, then complete the synthesis questions outside of class.

1. Consider the following two functions $f(x) = (x^2 - 2x) \cdot (x^3 + x^5)$ and $g(x) = x^2 \cdot e^x$. Is it possible to find the derivatives of these functions analytically using the rules that we have developed up to this point in the class? Explain your reasoning.

Recall: The Differential Operator: When you are given a function and you compute its derivative you are applying the *Differential Operator*. The Differential Operator , $\mathcal{D}$, is defined by:

$$\mathcal{D}(f) = f', \text{ for any differentiable function, } f(x)$$

Properties of the Differential Operator: In this activity we will investigate another property of the differential operator, $\mathcal{D}$, thus broadening again the class of functions that we can differentiate symbolically (i.e., find symbolic formulas for the derivative functions).

The Product Rule: This property concerns finding the derivative of the product of two functions. Let f and g be any two differentiable functions. We want to determine how $\mathcal{D}(f \cdot g)$ is related to $\mathcal{D}(f)$ and (g) .

2. *Experimentation*: Develop and carry out a thorough investigation to check which of the following is the correct derivative of $f(t) \cdot g(t)$. Remember that (analogously to previous investigations) you can use *Mathematica* to take the derivative of the product directly by first defining two functions, $f(t)$ and $g(t)$, and then defining the product, $h(t) = f(t) \cdot g(t)$.

   You will be graded on the investigation that you develop not on correctly identifying the derivative. Record the results of your investigation below.

   a. $f'(t) \cdot g'(t)$
   b. $f'(t) + g'(t)$
   c. $f(t) \cdot g'(t) + f'(t) \cdot g(t)$
   d. $f'(t) \cdot g(t) + g'(t)$

3. *Conjecture*: Express the relationship that you chose above in terms of the differential operator:

   $$\textbf{\textit{Product Rule}}: \quad \mathcal{D}(f \cdot g) = \_\_\_\_\_$$

4. More Practice with the Differential Operator:

   If $f(x) = 3x^6$ and $g(x) = (1.5)^x$ then: $\mathcal{D}(f \cdot g)$ = _____

Class Discussion: What Have We Learned/Recalled in this Activity?

Skills/Facts:

Methods:

Concepts to Understand:

Practice Applying the Product Rule: Find symbolic formulas for the following derivatives:

a. $f(x) = xe^x$ $f'(x) =$ _____

b. $f(x) = \cos x\,(3x^2 - 7)$ $f'(x) =$ _____

c. $f(x) = 3e^{5x}\sin(x)$ $f'(x) =$ _____

d. $f(x) = (x^3 + x^7)(x^{21} + x^4)$ $f'(x) =$ _____