MT 1800 Calculus I

Module II, CA 8: Exploration of Relationships between Exponential Functions and their Instantaneous Rate of Change

Name: ______________________________________

Purpose: To identify and utilize rate of change relationships exhibited in exponential functions.

Procedure: Work on the following activity with 1-2 other students during class (but be sure to complete your own copy) and finish the synthesis exploration before the next class.

1. Consider the concentration of theophylline, C (in mg/l), a common asthma drug, in the blood stream of a subject weighing 50 kg as a function of time, t (in hours after the initial dose), after the injection where $C(t) = 11.6(0.85)^t$.

   Calculate the concentration of theophylline, C, and the instantaneous rate of change of the concentration of theophylline, $\frac{dC}{dt}$, at the times listed below.

Time (*hours*), *t*	Concentration (*mg/l*), *C*	Instantaneous Rate of Change, $\frac{dC}{dt}$
1		
3		
5		
7		
9		
11		
13		
15		
17		
19		

2. Using Mathematica create the following scatterplots and attach them to this activity:
    a. The instantaneous rate of change of the concentration of theophylline, $\frac{dC}{dt}$, as a function of time, t.
    b. The instantaneous rate of change of the concentration of theophylline, $\frac{dC}{dt}$, as a function of the concentration of theophylline, C.

3. Find a modeling function for each of the following relationships and clearly state symbolically how the variables are related in each case.
    a. The instantaneous rate of change of the concentration of theophylline, $\frac{dC}{dt}$, as a function of time, t.

    b. The instantaneous rate of change of the concentration of theophylline, $\frac{dC}{dt}$, as a function of the concentration of theophylline, C.

<u>Synthesis Exploration</u>: After noticing a mathematical pattern or relationship in a specific example, our next step should always be to ask ourselves if this pattern or relationship is true more generally. We will follow our work in class today by asking ourselves exactly that.

4. Consider another exponential function $y = 1.7e^{2t}$. Conjecture what you think the relationship will be between the following pairs of variables.
   a. The instantaneous rate of change, $\frac{dy}{dt}$, and t.

   b. The instantaneous rate of change, $\frac{dy}{dt}$, and y.

5. Investigate whether your conjectures above are true using appropriate numerical calculations and graphs. Note below how your investigation leads you to believe or disbelieve your conjectures above.

   Investigation Notes:

Conclusions about your Conjectures:

a. The instantaneous rate of change, $\frac{dy}{dt}$, and t.

Conjecture:

Argument with Evidence for Believing or Disbelieving your Conjecture:

b. The instantaneous rate of change, $\frac{dy}{dt}$, and y.

Conjecture:

Argument with Evidence for Believing or Disbelieving your Conjecture:

6. Class Discussion: What relationships are true for the instantaneous rate of change (or derivative) of exponential functions?
   Narratively:

   Symbolically:

   Graphically:

We can refer to the relationships that you wrote down in #4, 5 and 6 above as **differential equations.** A differential equation expresses how the instantaneous rate of change (or derivative) of a changing quantity behaves. We can figure out quite a bit about the relationship between two changing quantities (say y and t) by just knowing how their instantaneous rate of change, $\frac{dy}{dt}$, behaves.

7. Using what we have concluded in #6, calculate the following derivatives:

   a. $y = e^{0.72t}$ $y' =$ ____________________

   b. $f(t) = (1.18)^t$ $f'(t) =$ ____________________

8. Class Discussion: We can write the process that we are using above as the following derivative rules for exponential functions:

   - If $f(t) = e^{kt}$ then $f'(t) =$ ___________________________
   - If $f(t) = a^t$ then $f'(t) =$ ___________________________

Class Discussion: What Have We Learned/Recalled in this Activity?

Skills/Facts:

Methods:

Concepts to Understand:

Skills/Methods Practice: Complete the following individually

1. Find symbolic formulas for the following derivatives.
   a. $y = e^{-3x}$ $\qquad \dfrac{dy}{dx} =$ ________________
   b. $y = e^{0.72t}$ $\qquad y' =$ ________________
   c. $y = (0.76)^x$ $\qquad \dfrac{dy}{dx} =$ ________________
   d. $y = 5^x$ $\qquad y' =$ ________________
   e. $f(t) = 3e^{5t}$ $\qquad f'(t) =$ ________________
   f. $f(x) = -2.7(1.09)^x$ $\qquad f'(x) =$ ________________
   g. $y = 2 \cdot 5^x + 7x^5$ $\qquad y' =$ ________________
   h. $y = 3^{2t}$ $\qquad \frac{dy}{dt} =$ ________________

2. Write down a differential equation showing the relationship between $\frac{dy}{dx}$ and y for the following functions.
   a. $y = e^{0.72x}$ $\qquad \dfrac{dy}{dx} =$ ________________
   b. $y = (0.76)^x$ $\qquad \dfrac{dy}{dx} =$ ________________
   c. $y = 5^x$ $\qquad \dfrac{dy}{dx} =$ ________________