Module II, CA 6: The Second Derivative and Its Applications
Synthesis Questions

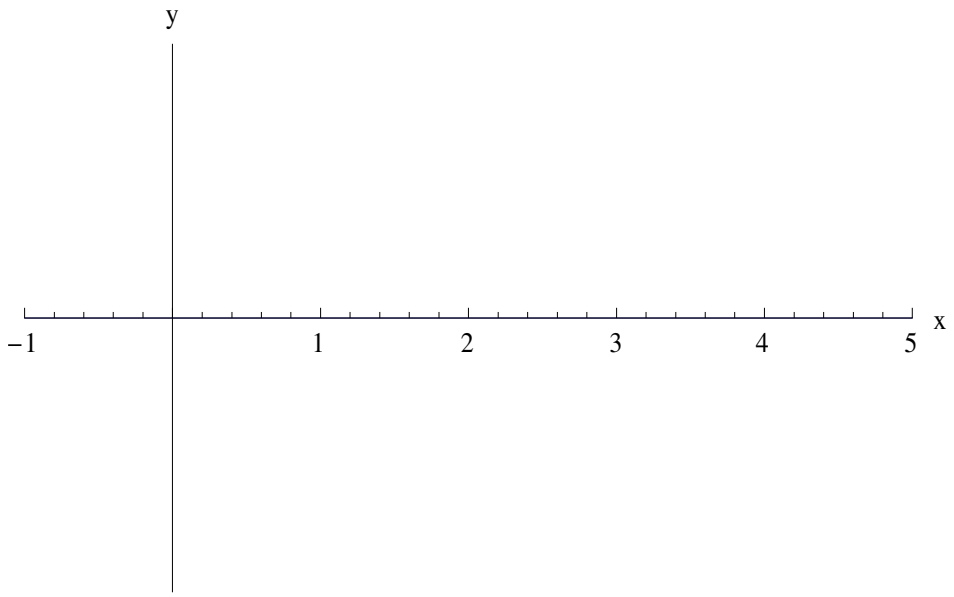
Name: _____

Purpose: To synthesize our work on the second derivative in class.

Procedure: Work on the following questions outside of class. You may consult with one or two other students. Each student should hand in their own copy of these synthesis questions.

1. Sketch the graph of a single function $f(x)$ that has the following characteristics:

x	$f(x)$	$f'(x)$	$f''(x)$
0	$-$	$+$	$-$
1	$-$	**0**	$-$
2	$-$	$-$	**0**
3	$-$	**0**	$+$
4	**0**	$+$	$+$
5	$+$	$+$	$+$



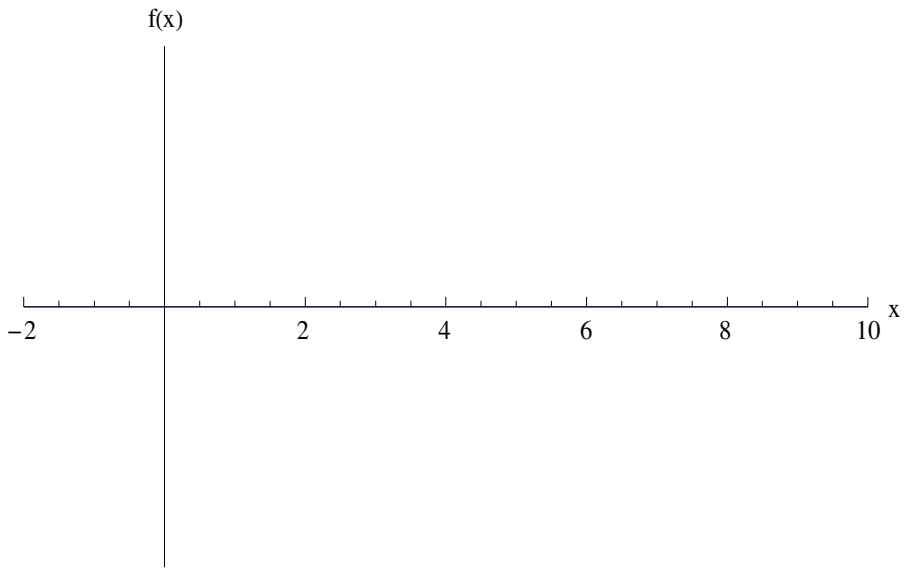
2. A function $f(x)$ has the following characteristics:

- Concave up in the interval $(-2,5)$
- Concave down in the interval $(5,10)$
- Second derivative is 0 at $x = 5$
- Has an instantaneous rate of change of 0 at $x = 2,$ and $x = 7$
- The instantaneous rate of change is positive between $x = 2$ and $x = 7$

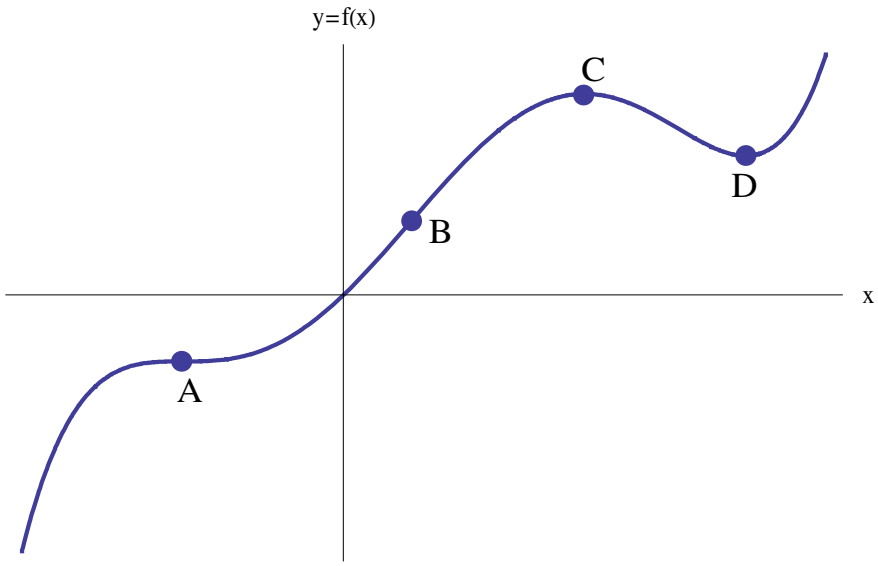
Circle all of the following statements that you definitely know to be true using only the above information:

a. $f(x)$ has a local minimum at $x = 7$
b. $f(x)$ is increasing from $x = 2$ to $x = 7$
c. $f(x)$ has a local maximum at $x = 5$
d. $f(x)$ has a local minimum at $x = 2$
e. $f(x)$ has an inflection point at $x = 5$
f. $f(x)$ has a local maximum at $x = 3$

Sketch a graph of a function that could be $f(x)$:



3. Consider the graph of a function $f(x)$ shown below. At each of the four points labeled on the graph, determine whether the first and second derivatives, $f'$and $f''$are positive (write "+"), negative (write "-"), or equal to zero (write "0").



Note: The function $f(x)$ graphed above has both a critical point and an inflection point at A, an inflection point at B, a local maximum at C, and a local minimum at D.

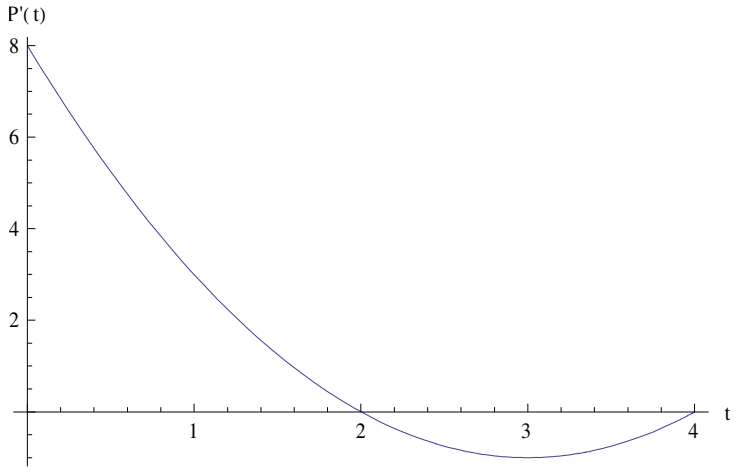
At the point A: f' is _____ and f'' is _____

At the point B: f' is _____ and f'' is _____

At the point C: f' is _____ and f'' is _____

At the point D: f' is _____ and f'' is _____

4. A patient is administered one painkiller at time $t = 0$ and over time after the painkiller was administered, t (in hours), the level of this painkiller in her bloodstream, $P(t)$, is monitored. The patient began with no painkiller in her bloodstream. The graph below shows the relationship between the instantaneous rate of change $(P'(t))$ of this level and time.



Over the course of the 4 hours when the level of painkiller was monitored:

a. When was the level of painkiller in the patient's body greatest?

b. When was the level of painkiller in the patient's body least?

c. When was the level of painkiller in the patient's body growing fastest?

d. When was the level of painkiller decreasing fastest?

e. On what interval is the graph of $P(t)$ concave up? Concave down?

f. If the patient takes another painkiller at time $t = 4$ what would the graph of $P'(t)$ look like from $t = 4$ to $t = 8$.