MT 1800 Calculus I

Module II, CA 2: The Derivative Function – A Graphical Approach

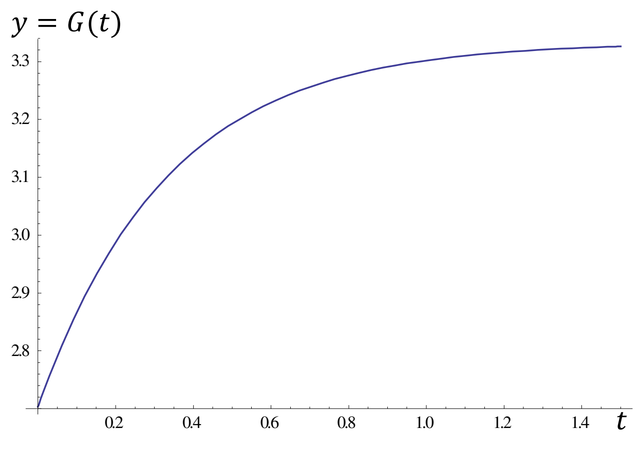
Name: ______________________________________

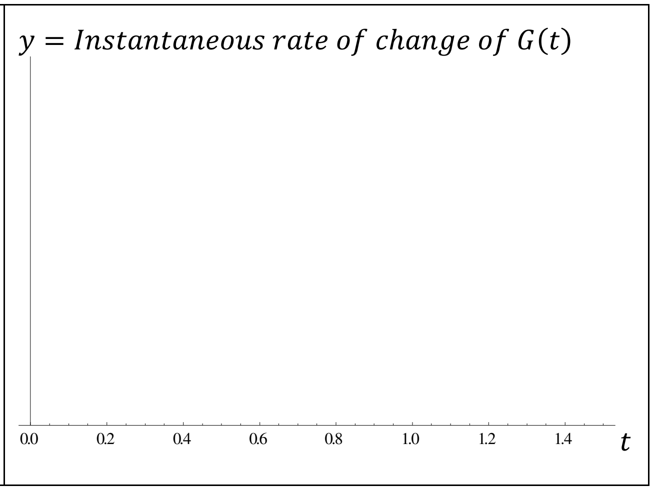
Purpose: To incorporate our understanding of instantaneous rates of change to build and apply the derivative function.

Procedure: Work on the following activity together in pairs, then complete the synthesis questions outside of class.

1. **The Instantaneous Rate of Change of Glucose:** Recall our glucose function, $G(t) = \frac{10}{3} - 0.63e^{-3t}$, giving the amount of glucose present in the bloodstream at time t hours after infusion begins.
   a. Estimate the instantaneous rate of change of $G(t)$ at $t = 1.4$.

   b. As a class, we have calculated very good estimates for the instantaneous rate of change of the amount of glucose in the bloodstream at a couple of different time values in CA II.1. Use these values together with your answer in a. and information from the graph of $G(t)$ (pictured below on the left) to sketch a graph of the ***instantaneous rate of change of the amount of glucose in the bloodstream as a function of time***.





Explain your reasoning in sketching the instantaneous rate of change curve above.

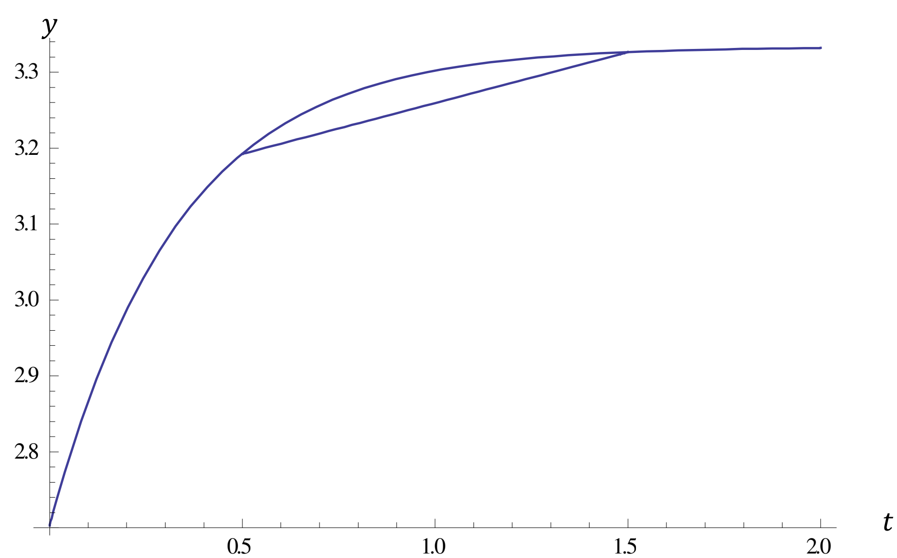
2. The **instantaneous rate of change of $G(t)$ is also called the derivative of $G(t)$** and is denoted as $G'(t)$. We also use the notation $\frac{dG}{dt}$ to represent the derivative of $G(t)$.
   a. Where do you think the $\frac{dG}{dt}$ notation comes from?

   b. The derivative, $G'(t)$, is a function – describe its input and output.

   c. The average rate of change of $G(t)$ between $t = 0.1$ and $t = 0.5$ is 0.815. What does this mean in the context of this glucose application?

   d. What does $G(0.5) = 3.193$ tell you about this glucose application? What does $G'(0.5) = 0.422$ tell you about this glucose application?

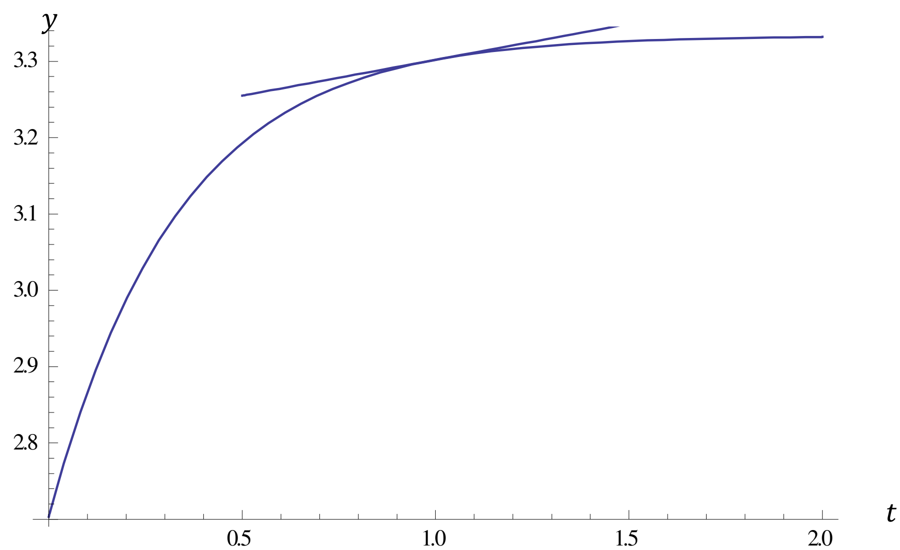
   e. On the graph of $G(t)$ below, the secant line between $t = 0.5$ and $t = 1.5$ is shown. Describe what it means to be a secant line in your own words.



f. Recall that you can define a single line by specifying its slope and one point it passes through. What slope must the secant line that you sketched in part (e) have? What points must this secant line pass through?

g. Write a formal definition for the secant line of $G(t)$ between $t = a$ and $t = b$.

h. On the graph of $G(t)$ below, the tangent line to the graph of $G(t)$ at $t = 1$ is shown. Describe what it means to be a tangent line in your own words.



i. Recall that you can define a single line by specifying its slope and one point it passes through. What slope must the tangent line that you sketched in part (f) have? What point must this tangent line pass through?

j. Write a formal definition for the tangent line of $G(t)$ at $t = a$.

k. Consider the tangent lines to $G(t)$ as t increases. How do these tangent lines relate to the graph of $G'(t)$?

3. **The Derivative Function:** Consider the polynomial function $f(x) = -2x^3 - 3x^2 + 36x + 12$.

a. Calculate the average rate of change of $f(x)$ between $x = 1$ and $x = 3$.

b. Calculate a good estimate for the derivative of $f(x)$ at $x = 1$. (Do your calculations in *Mathematica*.)

Derivative of $f(x)$ at $x = 1$ is approximately ____________.

c. Calculate a good estimate for the instantaneous rate of change of $f(x)$ at $x = 3$. (Do your calculations in *Mathematica*.)

Instantaneous rate of change of $f(x)$ at $x = 3$ is approximately ____________.

d. Sketch a graph of $f(x)$ between $x = 0$ and $x = 4$. On the same graph sketch, the secant line that connects the points on $f(x)$ at $x = 1$ and $x = 3$.

e. Find the equation of the secant line that connects the points on $f(x)$ at $x = 1$ and $x = 3$.

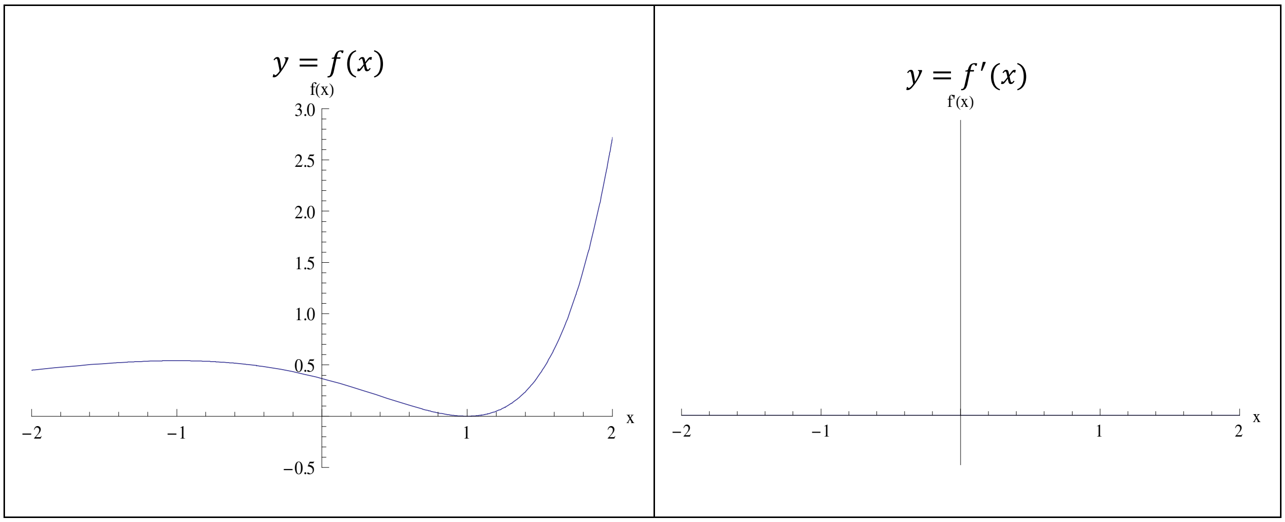
f. Sketch a graph of $f(x)$ between $x = 0$ and $x = 4$. On the same graph, sketch the tangent line to $f(x)$ at $x = 3$.

g. Find the equation of the tangent line to $f(x)$ at $x = 3$.

Equation of Tangent Line: ____

h. In *Mathematica*, graph $f(x)$, your secant line from part (e) and your tangent line from part (g) together on the same set of axes. Print out a copy of your graph and attach it to this activity.

4. Consider the graph of $f(x)$ provided below. On the axes on the right, sketch the graph of the derivative of $f(x)$ as a function of x. (You should sketch this graph using only the shape and behavior of the graph of $f(x)$ – you do not need an analytical form of $f(x)$ to do this.)



5. Class Discussion: How did you determine how to sketch $f'(x)$ above? What is a local maxima and local minima of a function?

6. At what x-values does the function $f(x)$ have ***local maxima*** or ***local minima***? What do you notice about $f'(x)$ at these values of x?

Class Discussion: What Have We Learned/Recalled in this Activity?

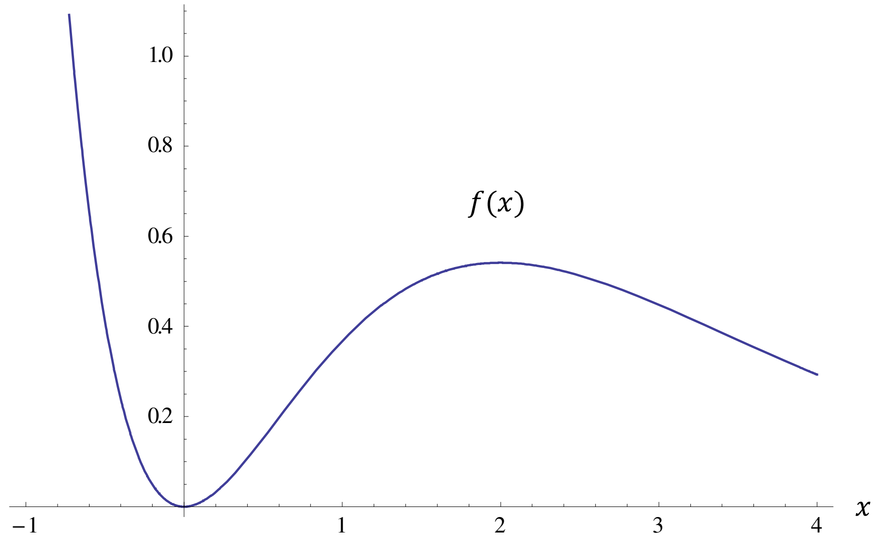
Skills/Facts:

Methods:

Concepts to Understand:

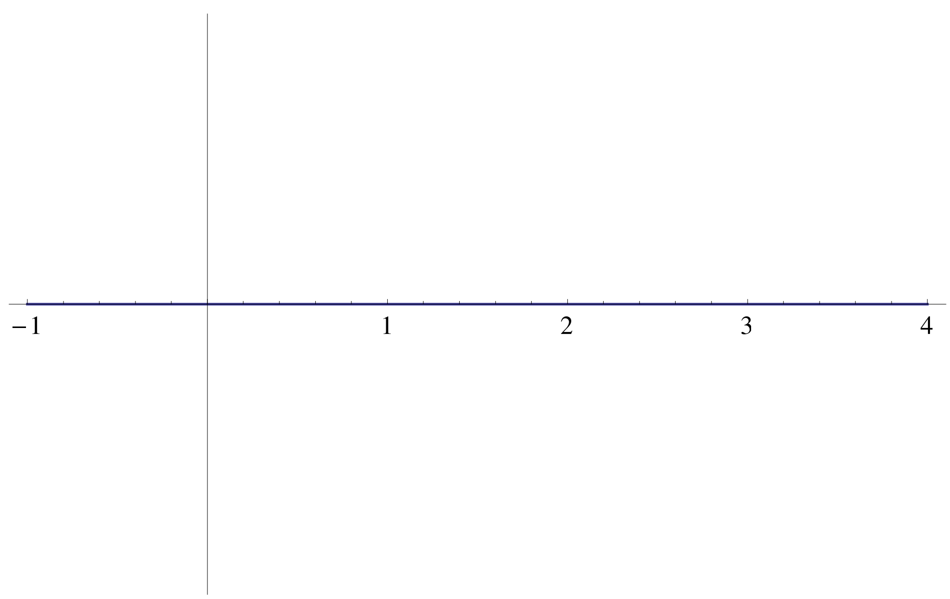
Skills/Methods Practice: Complete the following questions individually.

1. Consider the function $f(x)$ graphed below.



a. Sketch and clearly label the secant line of $f(x)$ from $x = 0$ to $x = 2$ on the graph above.
b. Sketch and clearly label the tangent line to $f(x)$ at $x = 2$ on the graph above.
c. At what x values does $f(x)$ have maxima and minima?

d. Sketch a graph of the derivative of $f(x)$ ($f'(x)$) in the space provided below. Circle the x values that correspond to the maxima and minima of $f(x)$.



2. Consider the function $f(x) = \frac{x^2}{e^x}$.
   a. Estimate the derivative of $f(x)$ at $x = 0.4$.

   b. Calculate $f(0.25)$.

   c. Estimate $f'(1.1)$.

   d. Estimate the instantaneous rate of change of $f(x)$ at $x = 0.25$.

e. Find and write the equation of the tangent line to $f(x)$ at $x = 0.25$.

f. Find and write the equation of the secant line of $f(x)$ between $x = 0.5$ and $x = 1.5$.