MT 1810 Calculus II
Module I, Concept Review Activity

Name: ___________________________________

Purpose: To get more practice with some concepts and to bring together related concepts, making final connections in preparation for the exam.

Procedure: Work on the following activity individually first to test your own understanding. Then compare with a classmate and discuss any differences in your answers. You will not need *Mathematica* or a graphing calculator to complete these questions, so you should not use them on this review activity (as they will not be necessary (or allowed) on the exam).

1. Which of the following integrals could be used to calculate the volume, in cubic feet, of a pyramid whose base is square with a width and length of 50 feet and whose altitude is 30 feet? Notice that at a height of h feet from the ground the pyramid has a width and length of $\frac{5}{3}(30-h)$ feet.

   a. $\int_0^{30} \pi\left(\frac{5}{6}(30-h)\right)^2 dh$

   b. $\int_0^{30} \left(\frac{5}{3}(30-h)\right)^2 h\, dh$

   c. $\int_0^{30} \left(\frac{5}{3}(30-h)\right)^2 dh$

   d. $\int_0^{30} \pi\left(\frac{5}{6}(30-h)\right)^2 h\, dh$

2. Consider the following expression:

$$\lim_{n\to\infty}\sum_{i=1}^{n} 4\left(\left(2+\frac{5i}{n}\right)^2 - 8\left(2+\frac{5i}{n}\right)\right)\left(\frac{5}{n}\right)$$

Choose the integral below that would give the same value as the mathematical expression above when calculated.

a. $\int_0^5 4(x^2 - 8x)\,dx$

b. $\int_2^8 4x^2 dx$

c. $\int_2^7 4(x^2 - 8x)dx$

d. $\int_2^7 4(x^2 - 8x)\left(\frac{1}{x}\right)dx$

e. $\int_2^5 4\big((2+x)^2 - 8(2+x)\big)dx$

3. Consider the following series:

$$\sum_{i=0}^{\infty} 5\left(\frac{1}{3}\right)^i$$

Circle all of the following statements that are true for this series:

a. The terms of this series decrease as i increases.

b. The second term of this sum is $\frac{5}{3}$.

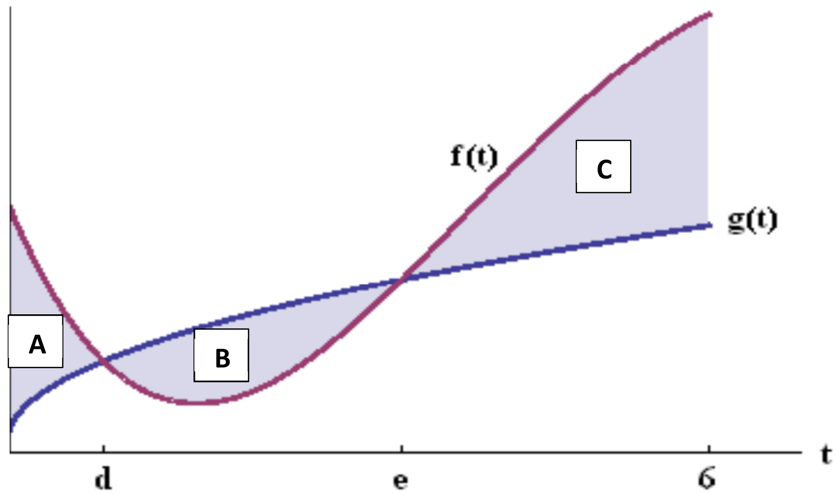
c. The series is equal to $\lim_{n\to\infty} 5\left(\frac{1}{3}\right)^n$.

d. Each term is one third the size of the previous term for $i \geq 1$.

e. The series is equal to $\lim_{n\to\infty}\sum_{i=0}^{n} 5\left(\frac{1}{3}\right)^i$.

f. The first three terms of the series are $\frac{5}{3}, \frac{5}{9}, \frac{5}{27}$.

4. Consider the following area between two curves, $f(t)$ and $g(t)$, graphed between $t = 0$ and $t = 6$:

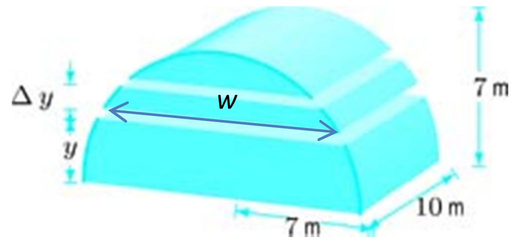


***Areas of Regions*: A = 6; B = 11; C = 25**

a. Consider each of the following definite integral statements. Circle all of the statements that could be true given the graph above.
   i. $\int_0^d f(t)dt = 10$, and $\int_0^d g(t)dt = 4$
   ii. $\int_0^e f(t)dt > \int_0^e g(t)dt$
   iii. $\int_0^6 f(t)dt < 31$
   iv. $\int_0^d f(t)dt > 0$
   v. $\int_0^6 f(t)dt > \int_0^6 g(t)dt$

b. Suppose $f(t)$ represents the speed (in mph) of Car A and $g(t)$ represents the speed (in mph) of Car B over a 6 hour car trip (t measured in hours). Circle all of the statements below that could be true given the graph above.
   i. Car A travelled farther than Car B between $t = d$ hours and $t = e$ hours.
   ii. At time $t = d$ hours, Car A and Car B have travelled the same distance.
   iii. Car A travelled 25 miles more than Car B between $t = e$ hours and $t = 6$ hours.
   iv. At time $t = d$ hours, Car A is reversing.
   v. Car B travelled exactly 11 miles between $t = d$ hours and $t = e$ hours.
   vi. If Car A and Car B started together on the same road for this car trip, then Car B is ahead of Car A for the entire time between $t = d$ hours to $t = e$ hours.
   vii. Car A is travelling faster than Car B for the entire time period between $t = 0$ hours and $t = d$ hours.
   viii. If Car A and Car B started together on the same road for this car trip, then after 6 hours Car A is 20 miles ahead of Car B.

5. Consider the following object:



We can use the Pythagorean Theorem to discover that at a height of y meters the width of the shape w is equal to $2\sqrt{49 - y^2}$ meters.

a. Estimate the volume of the object above using the slices shown. Use the following questions to help you with your estimate.
   i. What simplifying assumption would help you make an approximation?
   ii. What are the units of volume? What are the units of width and height?
   iii. Can you improve on your current estimate? How?

b. Write a mathematical expression for the exact volume of this object.

6. Answer the following questions about the convergence and divergence of series:
   a. For each of the following series you should now know if they converge without the use of any tests or *Mathematica*. For each of the series, write down whether the series converges or diverges.
      i. $\sum_{k=1}^{\infty} \frac{1}{k}$
      ii. $\sum_{k=1}^{\infty} \frac{1}{k^3}$
      iii. $\sum_{k=1}^{\infty} \frac{1}{k^{0.5}}$
      iv. $\sum_{k=1}^{\infty} \frac{1}{k^{1.25}}$
      v. $\sum_{k=0}^{\infty} 5(0.7)^k$
      vi. $\sum_{k=0}^{\infty} 0.4(1.5)^k$
      vii. $\sum_{k=0}^{\infty} 3(4)^k$

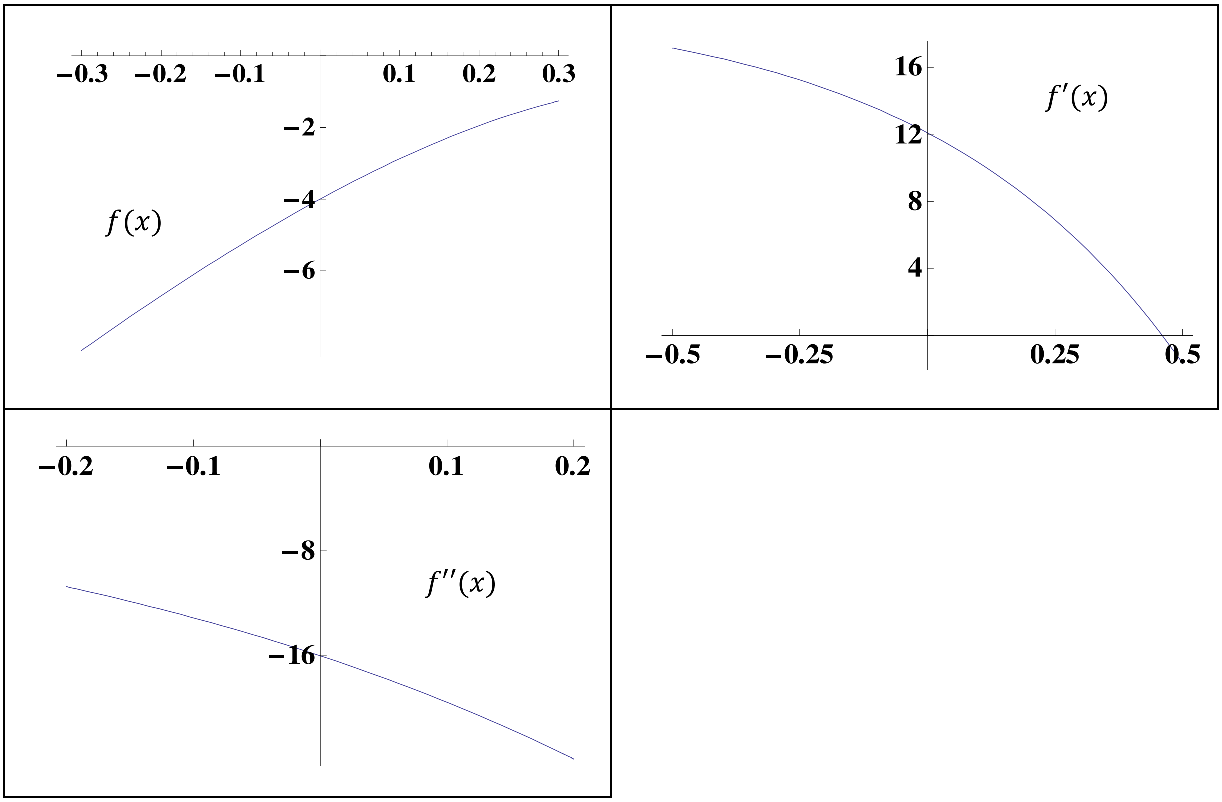
   b. For each of the series below decide whether each of the following tests allow us to decide if the series converges or diverges. If you are unable to use the test or it doesn't tell you anything explain why. For each of the series at least one of the tests will work.

	Ratio Test	Comparison Test	Divergence Test
i. $\sum_{k=0}^{\infty} \frac{3}{k!}$			
ii. $\sum_{k=1}^{\infty} \frac{(0.2)^k}{k}$			
iii. $\sum_{k=0}^{\infty} \frac{1}{k^2+7}$			
iv. $\sum_{n=0}^{\infty} \frac{2n}{n+1}$			
v. $\sum_{k=0}^{\infty} 2^k$			
vi. $\sum_{k=0}^{\infty} \frac{1}{k-7}$			
vii. $\sum_{k=0}^{\infty} \frac{(1.9)^k}{(k!)^2}$			

      viii. Do any of these tests help us determine the exact value that these series converge to?

   c. For some series we can determine the exact value that the series converges to. Calculate the exact value for the following infinite series. (Do not use *Mathematica* or your calculator for this question!)
      i. $\sum_{k=0}^{\infty} 5(0.1)^k$
      ii. $\sum_{k=0}^{\infty} 3.2(-0.1)^k$
      iii. $-\frac{6}{5} - \frac{12}{15} - \frac{24}{45} - \frac{48}{135} - \cdots$
      iv. $\sum_{k=1}^{\infty} 2(0.8)^k$

7. Consider the following graphs of a function, $f(x)$, the first derivative of the function and the second derivative of the function.



a. Use these graphs to write down a degree two Taylor polynomial for the function $f(x)$ about $x = 0$.

b. Find an approximate value for $f(-0.85)$. (Do not estimate this value by attempting to extend the graph above!)