Module III, Course Activity 1: Improper Integrals
Synthesis Questions

Name: ______________________________________

Procedure: Complete the following synthesis questions. Do your work on separate pages – provide all work, explanations, and circled answers on these separate pages. Do not use *Mathematica* to calculate any integrals.

1. In a genetics study, it was found that the distance, x, between mutations in a certain strand of DNA had a probability density function given by $f(x)$ where $f(x) = .2e^{-\frac{x}{5}}$ for all $x > 0$, and $f(x) = 0$ otherwise.

   a. What is the mean distance between mutations?

   b. What is the median distance between mutations?

   c. What is the probability that the distance between mutations will be greater than the mean?

   d. What is the probability that the distance between mutations will be greater than the median?

2. Imagine living in a large town where a number of people become initially infected with a flu (from a visitor to the town, perhaps). After this initial infection, the rate at which people are getting infected, $g(t)$, (measured in people per day) as a function of time t (measured in days since the initial infection) is given by:

$$g(t) = 40te^{-.05t}$$

   If we assume that this epidemic continues indefinitely, how many people will be infected in total?

3. In 1980, a coastal town noticed that their coastline was receding and so they decided to start measuring the rate of erosion while at the same time trying to combat this erosion by strengthening the coastline around the town. Their measurements showed that the rate at which the coastline was receding (beginning in 1981) t years after 1980 was approximately $\frac{1}{t}$ inches per year.

   a. If this model continues to hold true, how much will the coastline recede between the years 2080 and 2090?

   b. If this model continues to hold true, how much will the coastline recede in total after 1981?

   c. Do you think it's reasonable to assume this model will continue to hold true?

4. The ***Cauchy*** probability distribution has a probability density function given by:

$$f(x) = c * \frac{1}{1+x^2}, \quad -\infty < x < \infty$$

Note: For the questions below you will need to know the following facts about the function, $\arctan(x)$:

- $\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$
- Long-term behavior of $\arctan(x)$: $\lim_{x\to\infty} \arctan(x) =$ ______; $\lim_{x\to-\infty} \arctan(x) =$ ______

   a. Calculate what the value of the constant, c, must be in order for $f(x)$ to be a valid probability density function.

   b. The Cauchy probability distribution is known as a "pathological" distribution. Calculate the mean of this Cauchy distribution to see why.