MT 1810 Calculus II

Course Activity III.1: Improper Integrals

Name: ______________________________________

Purpose: To integrate functions with a discontinuity on the interval over which we are integrating and to calculate integrals where one of the bounds of integration is ∞ or $-\infty$.

Procedure: Work on the following activity with 1-2 other students during class (but be sure to complete your own copy) and finish the exploration outside of class.

Infinite Bounds of Integration

1. In MI CA2 Synthesis questions we considered the following questions:

   *Is the total area under the curve, $f(x) = \frac{1}{x}$ and to the right of $x = 1$ **finite or infinite**? Explain your thinking.*
   *Is the total area under the curve, $g(x) = \frac{1}{x^2}$ and to the right of $x = 1$ **finite or infinite**? Explain your thinking.*

   a. Recall your thinking on these two questions, express the area as an integral in each case and explain the process that you used to arrive at a final answer.

   b. <u>Class Discussion</u>: Using limit notation, write the process that you developed as a mathematical expression in each case.

c. Using the mathematical expression from (b), calculate both of the areas above analytically without using *Mathematica*. Show all of your work. Do you get the same answer as you did in MI CA2 Synthesis?

d. Class Conclusion: If $f(x)$ is a continuous function on the interval $[a, \infty]$ then we calculate the **improper integral** $\int_a^\infty f(x)dx$ in the following way:

e. Generalization: If $f(x)$ is a continuous function on the interval $[-\infty, a]$ then we calculate the **improper integral** $\int_{-\infty}^a f(x)dx$ in the following way:

2. ***Practice***: Decide whether each of the following integrals is finite or infinite. If the integral is finite, compute its exact value (analytically) **without the use of *Mathematica*** (although you should definitely use *Mathematica* to check your work!)

   a. $\int_1^\infty \frac{1}{\sqrt{x}} dx =$ ______________________________

   b. $\int_0^\infty \frac{1}{e^x} dx =$ ______________________________

   c. $\int_{-\infty}^{-3} \frac{1}{x^5} dx =$ ______________________________

   d. $\int_{-\infty}^{0} \frac{3}{e^x} dx =$ ______________________________

Unbounded Integrands:

3. Using the First Fundamental Theorem of Calculus, calculate the definite integral, $\int_0^3 \frac{2}{r^3}\,dr$. Are you able to calculate a value using the First Fundamental Theorem of Calculus? Is it appropriate to use the First Fundamental Theorem of Calculus in this situation? Explain your reasoning.

4. Using *Mathematica*, calculate the definite integral, $\int_0^3 \frac{2}{r^3}\,dr$. Explain any differences between your answer and *Mathematica's* answer.

5. Develop a process to *estimate* the integral $\int_0^3 \frac{2}{r^3}\,dr$ while avoiding the problem at $r = 0$. How could you improve on this estimate?

6. Using limit notation, write down a mathematical expression that incorporates the estimation process that you developed in (5) to represent an exact value for the integral.

7. Analytically evaluate the expression that you wrote down in (6) above to realize the answer that you received from *Mathematica* for this integral.

8. Repeat the investigation you carried out in (3)-(7) above for the integral $\int_0^3 \frac{2}{\sqrt{r}}\,dr$.

9. How does the integral $\int_0^3 \frac{2}{r^3}\,dr$ compare to the integral $\int_0^3 \frac{2}{\sqrt{r}}\,dr$? Explain what might have caused the differences and/or similarities. Why does the First Fundamental Theorem of Calculus **<u>appear</u>** to work for the integral $\int_0^3 \frac{2}{\sqrt{r}}\,dr$? (Note just because it appears to work does not mean we are justified in using it!)

10. Now consider the integral $\int_{-2}^{3}\frac{2}{r^3}$. Using the First Fundamental Theorem of Calculus, calculate the definite integral $\int_{-2}^{3}\frac{2}{r^3}$. Are you able to calculate a value using the First Fundamental Theorem of Calculus? Is it appropriate to use the First Fundamental Theorem of Calculus in this situation? Explain your reasoning.

11. Using *Mathematica*, calculate the definite integral, $\int_{-2}^{3}\frac{2}{r^3}\,dr$. Explain any differences between your answer and *Mathematica's* answer.

12. How should we approach evaluating this integral exactly without *Mathematica*? Describe your approach below and evaluate the integral.

Class Discussion: What Have We Learned/Recalled in this Activity?

Skills/Facts:

Methods:

Concepts to Understand:

Methods Practice: Decide whether each of the following integrals is finite or infinite. If the integral is finite, compute its exact value (analytically) **without the use of *Mathematica*** (although you should definitely use *Mathematica* to check your work!)

a. $\int_0^1 \frac{1}{x} dx =$ ____________________

b. $\int_{-4}^4 \frac{1}{x} dx =$ ____________________

c. $\int_{-2}^4 \frac{1}{\sqrt{x+2}} dx =$ ____________________

d. $\int_{-3}^3 \frac{1}{x^2} dx =$ ____________________