MT 1810 Calculus II

Course Activity I.2: Snowflake Exploration – Part II

Name: ___________________________________

Purpose: To further investigate shape characteristics of the Koch Snowflake. Introduction to finite sums and limits of finite sums (series), approximating exact limit with finite sums, and some error considerations.

Procedure: Work on the following activity with 1-2 other students during class (but be sure to complete your own copy). Complete synthesis questions outside of class. Save a copy of your *Desmos* work.

Practice with Sigma Notation:

$$\sum_{k=1}^{4} k = 1 + 2 + 3 + 4 = 10$$

$$\sum_{k=1}^{5} 2k^2 = 2 \cdot 1^2 + 2 \cdot 2^2 + 2 \cdot 3^2 + 2 \cdot 4^2 + 2 \cdot 5^2 =$$

$$\sum_{k=1}^{3} k(k+1) =$$

$$\sum_{k=0}^{4} \frac{3^k}{2} =$$

Write the following sums using sigma notation:

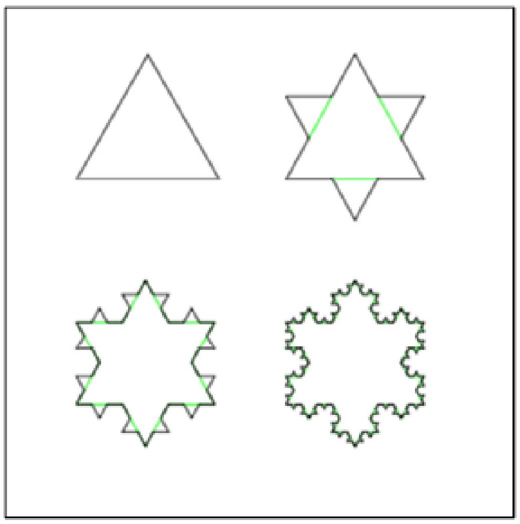
$$3 + 6 + 9 + 12 = \sum$$

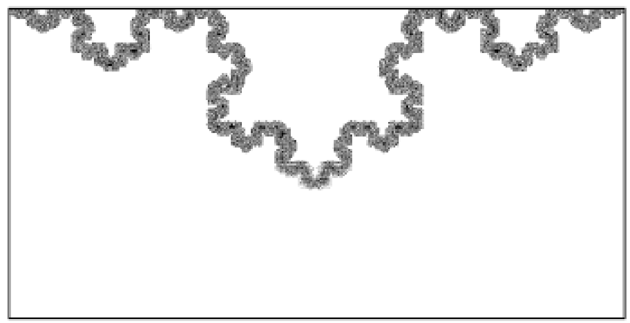
$$1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} = \sum$$

$$1 + \frac{3}{4} + \frac{4}{8} + \frac{5}{16} + \frac{6}{32} = \sum$$

Desmos has a sigma summation in the keypad (bottom left corner) under the **Functions** button at the bottom of the **Misc** tab. Use this to check that you have written the sums above correctly in sigma notation.

<u>The Koch Snowflake</u>: The objects that you have been recursively constructing are called **Koch Snowflakes**, named after the Swedish mathematician who first studied them, Niels Fabian Helge von Koch (1870 – 1924). The starting triangle and the first three iterations of the snowflake are shown in the figure on the left below. These mathematical shapes are stages leading to the *Koch curve*, one of the earliest fractal curves to be described. If the recursive process that you went through in Part I of this activity was carried out infinitely, the resulting object would be the Koch curve. A section of the Koch curve is shown to the right below. When magnified, it has an infinitely repeating self-similarity. (This characteristic is a feature of all fractals.)





1. Write an expression for the area of Snowflake $n = 4$ using sigma notation. Using Desmos, check this expression against the area that you have already found for Snowflake $n = 4$ in the last activity.

2. Calculate the area of Snowflake $n = 20$.

3. Use the patterns that you recognized in Part I of this activity to write a formula for the area of the n^{th} Koch Snowflake using sigma notation:

$A(n) =$

4. Class Discussion: As described above, the Koch curve is the limit of an infinite recursive process - where in each iteration, additional area is added to the previous snowflake. Do you think the area enclosed by the Koch curve is finite or infinite? Explain your reasoning and be ready to share this with the class.

5. *Desmos* Investigation: With the help of *Desmos*, we will numerically investigate the question of whether the Koch curve encloses an infinite or finite amount of area. Now that you have developed a formula for the area of the n^{th} Koch Snowflake, you can use a computer to easily calculate the area of any Koch snowflake.

   Calculate several areas to investigate the question posed earlier: Is the area enclosed by the Koch curve finite or infinite? After you have completed your investigation in *Desmos*, state your conjecture below and answer the questions that follow.

   a. Conjecture:

   b. Explain how your results in *Desmos* support your conjecture:

   c. If you have conjectured that the area enclosed by the Koch curve is finite, give an estimated value for the exact area enclosed by the Koch curve. What did you do to obtain this estimate?

6. Class Discussion: Exact Area Enclosed by the Koch Curve: You now realize that the area enclosed by the Koch curve is a series – the sum of an infinite number of terms. Every series is defined as the limiting value of the sequence of partial sums – here the partial sums are the increasing areas of the stages of the Koch Snowflake. Write the area enclosed by the Koch curve using a series in sigma notation:

   Area enclosed by the Koch curve = $\sum$

7. Approximations and Error:
   Use *Wolfram Alpha* to calculate the exact value for the expression above representing the area enclosed by the Koch curve. (For an example of using Wolfram Alpha to compute an infinite sum see https://www.wolframalpha.com/input/?i=sum+%283%2F4%29%5Ej%2C+j%3D0..infinity&lk=3).

   a. In Q5 you have calculated partial sum approximations for the exact area enclosed by the Koch curve – these are the areas of iterations of the Koch Snowflake. Use the $n = 5$, $n = 10$ and $n = 50$ Koch snowflakes as your partial sum approximations and for each approximation, calculate the associated error using *Desmos*. (Error = exact value – approximation)

   b. Suppose we need a partial sum approximation for the exact area enclosed by the Koch curve that is within $.00001 = 10^{-5}$ of the exact area. What is the smallest n for which the area of the n^{th} Koch snowflake will give us a sufficiently accurate approximation? Explain your reasoning.

Class Discussion: What Have We Learned/Recalled in this Activity?

Skills/Facts:

Methods:

Concepts to Understand:

Skills/Facts Practice: Complete individually.

1. $\sum_{k=1}^{4} k(k^2+1) =$

2. $\frac{1}{3}+\frac{1}{6}+\frac{1}{12}+\frac{1}{24}=\sum$

Methods Practice: Complete individually.

1. Write a formula for the area of the 30th Koch Snowflake ($n = 30$) using sigma notation.

2. Calculate the area of the 30th Koch Snowflake ($n = 30$). (You may use *Desmos*.)

3. If we use the area of the 30th Koch Snowflake as an approximation for the area enclosed by the Koch curve, what is the error associated with this approximation? (You may use *Desmos*.)

You are now ready to start the **Synthesis Questions** that can be found at https://sites.google.com/site/studentcalculusii/